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# Influence of pick time distribution on expected throughput of dual-tray VLMs

*The efficiency of an order-picking operation both in manufacturing and distribution environment can be enhanced with AS/RSs, which can provide an alternative picker-to-parts solution for small objects picking from small racks where the products are stocked in cartons, totes, boxes or on trays. One of them is Vertical Lift Module (VLM) in which insertion/extraction (I/E) device is traveling vertically and extracts trays or totes from the shelves and brings them to the operator putting it on pick shelf. While usual VLM systems have only one picking place, recently some producers of VLMs offer solution with two pick places, naming it dual-tray VLM or dual-bay VLM. To design order-picking systems with dual-tray VLM, analytical throughput model is developed. However, model assumes pick time by human operator either deterministic or exponentially distributed. So in this paper different pick time distributions are analysed using simulations and compared with analytical solutions.*

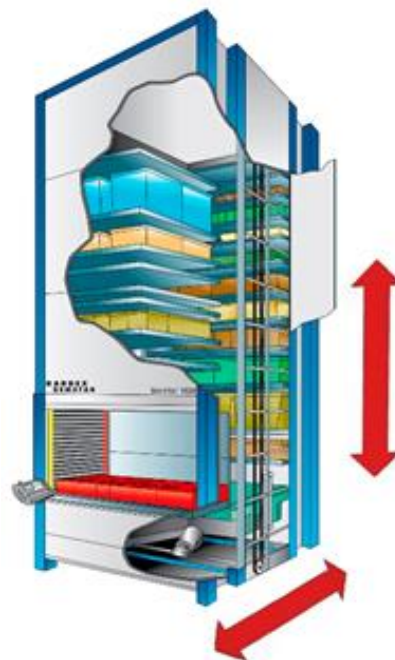
**Keywords:** Vertical lift module systems, Throughput model, Order-picking, Pick time distribution.

## 1. INTRODUCTION

Automated storage and retrieval systems (AS/RS) have been widely used in manufacturing and warehousing/distribution environments for a more than half a century. Despite higher investments costs and less flexibility, the usage of AS/RSs provide several advantages over non-automated systems, like labour costs and floor space savings, increased reliability and better accuracy [1]. Those advantages create benefits of using AS/RSs both for storage and order-picking operations in warehouses. Order-picking process is the most laborious and the costliest activity in a typical warehouse. With up to 55% of warehouse total operating costs [2], it is obvious why many companies are improving their order picking operations by using more efficient systems. In the case of a traditional low-level picker-to-parts warehouse, the items to be picked are positioned on the lower stocking locations of the shelves. The pickers usually use electric pallet trucks to move along the aisles and to transport one or more mixed pallets, composed of the items collected during their order picking activity [3]. Since in those cases traveling amounts for around 50% of total picking time [2], the logical way of improving it is to reduce or eliminate unproductive walking time.

The efficiency of an order-picking operation both in manufacturing and distribution environment can be enhanced with AS/RSs, which can provide an alternative picker-to-parts solution for small objects picking from small racks where the products are stocked in cartons, totes, boxes or on trays. Also called dynamic solutions, such systems as vertical carousels, horizontal

carousels, vertical lift modules, mini-load AS/RS systems, A-frames and picking machines, as well as the robots that have been recently employed, can assure higher space utilization and reduced travel distances [4]. In the focus of this paper is Vertical Lift Module (VLM), illustrated in Figure 1, in which insertion/extraction (I/E) device is traveling vertically and extracts trays or totes from the shelves and brings them to the operator putting it on pick shelf (or pick window) [5,6].



**Figure 1. Vertical Lift Module (VLM) with motion directions of I/E device**

Usual VLM systems have only one picking place, however recently some producers of VLMs offer solution with two pick places, naming it dual-tray VLM

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or dual-bay VLM. With the appearance of dual-tray VLM, logical consequence was to develop throughput model for it, as an aid to warehouse designers and managers in determination of expected throughput. Such model was developed in [7]. However, developed analytical model assumes pick time by human operator either deterministic or exponentially distributed. It is unlikely that pick time in practice will be exactly deterministic or stochastic with exponential distribution, more likely it will follow some other theoretical distribution or adequate empirical distribution.

Analytical throughput model for dual-tray VLM with human picker is shortly presented in next section. Developed simulation model used to simulate closed-loop system with VLM extractor and human picker as servers is presented in section 3, together with the results obtained by applying various distributions of service time of pickers. Last section gives conclusions.

## 2. ANALYTICAL THROUGHPUT MODEL FOR DUAL-TRAY VLM WITH HUMAN PICKER

While literature addressed models to design most types of AS/RSs, dual-tray VLMs are still relatively new in this arena. Not many papers deal with the VLM systems although they are in use in practice since early 1970's, both in warehouse and manufacturing applications. The most important paper that presented throughput model for single-tray VLM with human order-picker is [8].

Based on that model and models of mini-load AS/RS [9, 10], the throughput model for dual-tray VLM is presented in [7]. This model assumes single VLM device with one crane and two pick places (for two trays in VLM's window), one above another, as illustrated schematically in Figure 2.

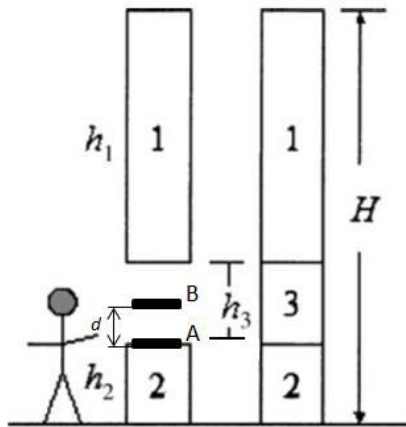


Figure 2. Side view of dual-tray VLM with typical sections, taken from [7]

While human picker is picking item(s) from one tray, I/E crane of VLM is able to store previous tray and retrieve next tray. Storage and retrieval of trays is done by I/E crane doing dual commands, alternatively from lower and upper pick position. The throughput model is based on cycle of the system. I/E device is doing dual command cycles (storing previous tray and delivering next one in one such cycle), while expected dual command cycle time could be calculated as average of

two expected dual command time based on models from [8] (for more details please refer to the [7]).

With the similarity of dual-tray VLM operating characteristics and mini-load AS/RS operating characteristics, the same idea for used to develop throughput model. Due to complex form of the probability distribution for dual command cycles, storage/retrieval travel time was approximated with a uniform distribution. Limits of uniform distribution of expected dual command travel time depend on expected dual command travel time  $E(DC)$  and its standard deviation  $S(DC)$  (which is presented as the fraction of dual command travel time and could be approximated as  $S(DC) \approx 0.383 E(DC)$ ). So the limits of uniform distribution of dual command cycle time are

$$\begin{aligned} t_1 &= E(DC) - 1.7321 \cdot S(DC) + C \\ t_2 &= E(DC) + 1.7321 \cdot S(DC) + C \end{aligned} \quad (1)$$

where  $C$  is constant tray handling time per dual command cycle.  $C$  consists of four times to pickup and deposit a tray (two pickups and two deposits) and four times to accelerate and decelerate.

Pick time was assumed either deterministic or exponentially distributed. If the pick time per delivered tray  $p_T$  is deterministic, expected system cycle time is

$$E(CT) = \begin{cases} E(DC) + C & \text{for } 0 < p_T \leq t_1 \\ \frac{p_T^2 - 2p_T t_1 + t_2^2}{2(t_2 - t_1)} & \text{for } t_1 < p_T \leq t_2 \\ p_T & \text{for } t_2 < p_T \end{cases} \quad (2)$$

while in case pick time is exponentially distributed, expected system cycle time is

$$E(CT) = E(DC) + C + \frac{p_T^2}{t_2 - t_1} [\exp(-t_1 / p_T) - \exp(-t_2 / p_T)] \quad (3)$$

From the calculated expected system cycle time one can calculate expected picker's utilization  $E(PU)$  and system's throughput  $R_T$  as

$$E(PU) = p_T / E(CT) \quad (4)$$

$$R_T = 3600 / E(CT) \quad \text{or} \quad R_T = 3600 \cdot E(PU) / p_T \quad (5)$$

Accuracy of analytical model was tested using comparison with results obtained by simulation models, confirming proposed model satisfactory accurate for estimating throughput of system with single dual-tray VLM and human order-picker.

Apart from papers presenting above mentioned models, only few more papers are dealing with VLMs. In [11] simulation-based approach to estimate the performance of single-bay VLM is presented for various configurations. In [12] authors focus on order batching optimization, considering single-tray VLM but with assumed constant pick time and constant storage/retrieval time. In [13] authors are considering dual-tray VLM order picking system under different configurations. Unlike in previous models with assumed

random storage policy, they are considering different storage policies and retrieval sequencing policies. The same authors in [14] compare dual-bay VLM to a carton racks warehouse. Those two storage solutions are analysed from an economic and a performance point of view.

### 3. SIMULATION MODEL FOR DUAL-TRAY VLM WITH HUMAN PICKER AND RESULTS

Simulation model was build in Enterprise Dynamics 10.2 simulation software. It is simulation software tool for 2D and 3D simulation modelling of discrete-event simulations using so-called atoms as entities. In this case system with dual-tray VLM and human picker was modelled as closed, two server cycling system with trays as customers, circulating in the loop and served alternately by the human picker and VLM's I/E device. Model with its entities and connections is shown in Figure 3.

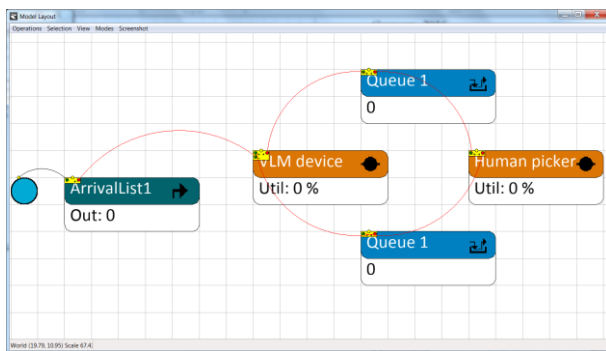


Figure 3. Simulation model of dual-tray VLM with human picker

VLM device and Human picker are server entities representing the system. Two “customers” (entities *Product* created at the start) are circulating in the loop between servers. Two queue entities are representing both A and B places of trays, where either delivered tray is waiting on human picker (*Queue 1*) or another tray is waiting on I/E device (*Queue 2*) to be taken back into shelving position.

Important attributes of servers are cycle times (service times), being uniform distribution of expected dual command cycle time of VLM device (therefore “customer” being served by this server represent actually two trays – one being taken back to the shelf and another being retrieved from the shelf and delivered to the pick window) and selected distribution of expected pick time per tray of Human picker. Capacity of queues is set to one. Utilisation of the server Human picker corresponds to the expected picker’s utilization  $E(PU)$  from Eq. (4), which value is then simply used to calculate expected system cycle time and corresponding system’s throughput using Eq. (5).

Simulation model was verified using deterministic service times and observations in real-time using software’s animation, while validated comparing results for deterministic and exponentially distributed pick times for known configurations presented in [7]. Those configurations are with 4 different heights of VLM device ( $H$  in mm), 3 different speeds of I/E device ( $v$  in cm/s) and three different average pick times per tray for

each configuration ( $p_T$  in s). Values of pick time are corresponding to the lower limit, upper limit and average value of uniformly distributed dual command cycle of VLM. Constant  $C$  was set to 24 seconds. Calculated expected dual command cycle times used to determine uniformly distributed cycle time of VLM device are given in Table 1.

Table 1. Analytical results for dual-tray VLM expected dual command cycle times,  $E(DC) + C$ , (in seconds)

$H$ (mm)	$v=50$ cm/s	$v=100$ cm/s	$v=150$ cm/s
4500	33.88	28.94	27.29
6000	37.64	30.82	28.55
7500	41.50	32.75	29.83
9000	45.40	34.70	31.13

Average service time of Human picker was selected from the Table 2 and used with different distributions for the purposes of validation and analysis.

Analysis of different pick time distributions on expected dual-tray VLM throughput was done with arbitrary chosen two uniform distribution of pick time (one with small and one with large range) and 3 normal distributions (with small, medium and large standard deviation).

Results are presented in Table 3 only for the case of VLM with  $H=7500$  mm due to the clarity of the presentation, while results for other VLM heights are quite similar in terms of conclusions.

### 4. CONCLUSION

In this paper different distributions of pick time in dual-tray VLM with human order-picker were analyzed, in order to see influence of different distributions (and deviations) on resulting throughput, in the same time compared to the previously available analytical solutions for deterministic and exponentially distributed pick times only.

Simulation analysis showed that expected system cycle time (therefore resulting throughput) depends on the pick time distribution. Results for distributions with smaller deviations are closer to the analytical deterministic solutions, while distributions with larger deviations are getting closer to the analytical exponential distribution. This was of course expected, however one should be aware of real practice pick time distribution and it’s influence on the resulting throughput.

Simulation results presented here contribute to the understanding and design of order-picking with dual-tray VLM systems in practice. In the further research more empirical distributions (possibly taken from practice) will be tested.

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**Table 2. Average pick time per delivered tray for various tested VLM configurations (in seconds)**

$H$ [mm]	$v$ [cm/s]								
	50			100			150		
	$p_{T1}$	$p_{T2}$	$p_{T3}$	$p_{T1}$	$p_{T2}$	$p_{T3}$	$p_{T1}$	$p_{T2}$	$p_{T3}$
4500	27	34	41	25	29	33	25	27.5	30
6000	28	37.5	47	26	31	36	25	28.5	32
7500	29	41.5	54	26	32.5	39	25	29.5	34
9000	31	45.5	60	27	34.5	42	26	31	36

**Table 3. Average system cycle times by simulation model and analytical model for different distributions of pick time per tray (in seconds)**

$H$ [mm]	Pick time distribution	$v$ [cm/s]								
		50			100			150		
		$E(CT)_1$	$E(CT)_2$	$E(CT)_3$	$E(CT)_1$	$E(CT)_2$	$E(CT)_3$	$E(CT)_1$	$E(CT)_2$	$E(CT)_3$
7500	Deterministic, analytical	41.50	44.40	54.00	32.75	34.08	39.00	29.83	30.64	34.00
	Deterministic, simulation	41.49	44.39	54.00	32.75	34.10	39.00	29.83	30.63	34.00
	Uniform (0.75 $pT$ , 1.25 $pT$ )	43.74	48.71	56.60	32.91	35.02	39.59	30.08	31.69	34.69
	Uniform (0.25 $pT$ , 1.75 $pT$ )	44.96	51.30	60.40	34.99	38.83	43.62	32.51	35.24	38.51
	Normal ( $pT$ , 0.1 $pT$ )	43.61	48.54	56.31	32.83	34.54	39.27	29.94	31.15	34.31
	Normal ( $pT$ , 0.5 $pT$ )	45.17	51.81	61.02	35.28	39.11	43.97	32.72	35.46	38.77
	Normal ( $pT$ , $pT$ )	45.60	54.75	65.30	37.09	42.15	47.74	34.72	37.87	42.08
	Exponential, simulation	48.66	56.93	66.75	40.19	44.64	49.48	37.43	40.58	43.98
	Exponential, analytical	48.62	56.96	66.73	40.19	44.68	49.65	37.44	40.60	44.00