A Queuing Network Approach for Performance Estimation of Shuttle Based Storage and Retrieval System Design

Banu Y. Ekren
Associate Professor
Yasar University
Faculty of Engineering, Dept. of Industrial Engineering, Izmir Turkey

Anıl Akpunar
Fellow (Research Assistant)
Yasar University
Faculty of Engineering, Dept. of Industrial Engineering, Izmir Turkey

Zaki Sari
Professor
University of Tlemcen
Manufacturing Engineering Laboratory of Tlemcen (MELT), Algeria

Tone Lerher
Professor
University of Maribor
Faculty of Mechanical Engineering, Maribor, Slovenia

Shuttle-based storage and retrieval system (SBS/RS) is relatively a new automated storage and retrieval technology developed as an alternative to the mini-load crane-based automated storage and retrieval system (AS/RS). It is important to have analytical models calculating critical performance measures promptly facilitating selection of the right warehouse design meeting the requirements of the company. The aim of this study is to present an open queuing network (OQN) model that can estimate the critical performance measures of a pre-defined SBS/RS design. The estiamated performance measures are considered to be the mean waiting time of a transaction in the queues, as well as the mean utilization of the servers and the mean number of transactions waiting in the server of queues. By the provided model, one would be able to evaluate an SBS/RS' design promptly, by changing those design criteria. The analytical model's results are validated by the simulation results by considering Absolute Percentage Error .

Keywords: *SBS/RS, open queuing network, performance calculation, automated warehousing*

1. INTRODUCTION (ALL CAPS - Helvetica 9 pt, bold) - Align left

Recent advances in warehousing automation technology have created various types of storage and retrieval systems. One of them is shuttle-based storage and retrieval system (SBS/RS) that is developed to cope with high transaction rate ([1]-[11], [14]). It is critical for companies to decide on the right technology with a right design of it for its business requirements. Therefore, the development of analytical models producing critical performance measures from those systems are critical to evaluate such systems' performance promptly. In this study, our aim is to develop such an analytical model, specifically OQN, estimating several critical performance measures from a pre-defined SBS/RS design.

2. SBS/RS DESCRIPTION AND ASSUMPTIONS FOR MODELLING

Figure 1 shows the top view of a tier of an aisle of the studied SBS/RS. There are two buffer areas in each tier where loads are dropped-off to be picked up by the lifts or shuttles. Namely, the lift drops-off the load on the buffer location when the transaction is a storage; picks-up the load when the transaction is a retrieval. Hence, there are two types of transaction requests: storage and retrieval. The processes take place based on these transactions are summarized below:

Storage Transaction: 1- lift moves from its dwell point to the ground-floor tier, i.e., the first tier – the I/O point of the system. 2- the lift picks up the storage load and travels to the designated tier. 3- when the lift reaches its destination, it releases the load in one of the two buffer locations. 4- the shuttle in the designated tier moves from its dwell point to the buffer location. 5- the shuttle picks-up the load. 6- the shuttle travels to the designated storage address with the load and releases it in the storage location.

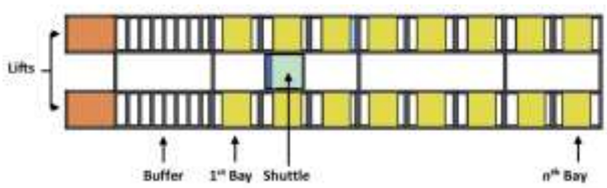

Figure 1: Top view of an SBS/RS

Retrieval Transaction: 1- the shuttle in the designated tier moves from its dwell point to the retrieval address to pick-up the load, and then travels to the buffer location. 2- the shuttle releases the load in one of the buffer locations. 3- the lift moves from its dwell point to the designated tier. 4- the lift picks up the load from the buffer location. 5- the lift travels to the I/O point (first tier) with the load and releases it.

The assumptions that are used in the SBS/RS modelling (both in analytical and simulation models) are summarized below:

- Transactions are discharged at any one of two buffer locations randomly to be picked up by the lift/shuttle.
- The lift and shuttles follow the single command cycle (SC) scheduling rule.
- The travel distance from the first bay to the buffer location (DB) is assumed to be same as the travel distance between two adjacent bays.
- Loading and unloading delays are ignored in the models.
- The dwell point of lifts/shuttles are assumed to be the points where they complete their last process.
- A pure random storage policy is assumed in the models. Under this policy, the storage address is assigned randomly by selecting any tier and bay with probability $1/T$ and $1/B$, respectively

(where T: the number of tiers; B: number of bays).

- In travel time estimations, acceleration and deceleration delays are considered to be the same ($a_s = d_s$ and $a_L = d_L$).
- If the transaction is at the first tier, then lift is not utilized.
- The storage and retrieval transaction arrivals follow independent Poisson distributions whose mean rates are equal, $\lambda_S = \lambda_R$.
- The distance between two adjacent bays and tiers are assumed to be 0.5m. and 0.35m., respectively.

3. OPEN QUEUING NETWORK MODELLING OF THE SBS/RS

The SBS/RS queuing system can be modeled as an OQN. In the OQN model of an SBS/RS, storage and retrieval transactions are assumed as customers and, the lifts and shuttles are two different types of servers. Fig. 2 shows an OQN model of the studied SBS/RS. An arriving transaction (storage or retrieval) enters the network of servers immediately. λ_S shows the mean arrival rate of the storage transactions and λ_R shows the mean arrival rate of the retrieval transactions in the system. Nodes represent the servers (i.e., lifts and shuttles). Note that the storage transactions enter the system from the lift node while the retrieval transactions enter the system from the shuttle node.

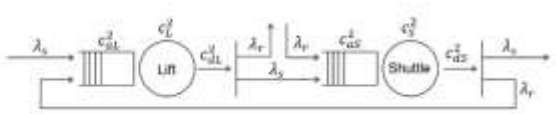

Figure 2: Open queuing network of the studied SBS/RS

Due to the random storage policy, by the assumption that each aisle, we model a single aile in the system. Based on the decomposition approach explained in the following sections, the first node can be modeled as a $G/G/2$ queuing system. In this queuing system, the lift's capacity is doubled and the arrival and the service rates are assumed to be generally distributed. The second node can be modeled as a $G/G/m$ queuing system where there is an m number of shuttles in an aisle (i.e. $m = T$). Note that a generally distribution can be described by their first two moments – the mean and the squared coefficient of variation (scv - c^2). Scv is the ratio of variance (σ^2) to the mean square (μ^2) [12], [13]. In Figure 2, each node represents a service delay represented by the two moments μ and c^2.

In calculating each node's μ and c^2 values, we utilized expected value calculations and Discrete Time Markov Chain (DTMC) modelling approach [4].

Correspondence to: Dr. Banu Y. Ekren, associate professor
Faculty of Engineering, Dept. of Industrial Engineering
Yasar University, Agacli Yol, Bornova Izmir, 35030 Turkey
E-mail: banu.ekren@yasar.edu.tr

3.1 Qeueing performance measure calculations

In Figure 2, the queuing network of the studied SBS/RS is shown. Remember that the nodes in that figure represent the servers: lifts and shuttles. Also remember that the service times of lifts and shuttles are represented by two moments, mean rate (μ) and squared coefficient of variation (c^2). Based on that figure, three basic network operations: departure, split and superposition on arrival rates take place. The first and the second moment calculations of these network operations are summarized by Figures 3-5, respectively [13]. For instance, Figure 3, 4 and 5 show the departure, the split and the superposition network operations and their two moment calculations. After calculating these first and second moments, we compute the queuing performance measures: the mean waiting time of a transaction in a lift queue ($E(W_L)$) as well as the mean number of transactions waiting in a lift queue ($E(L_L)$). We also compute the server utilizations namely, the utilization of lifts and the shuttles, ρ_L and ρ_S.

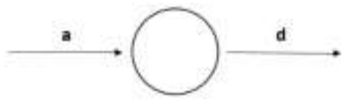

Figure 3: Departure network operation

In Figure 3, "a" shows arrival and "d" shows departure. This figure is a departure network operation. Based on that the departure's rate and the scv are calculated by (1)-(2) [12]:

$$\lambda_d = \lambda_a \quad (1)$$

$$c_d^2 = 1 + (1-\rho^2)(c_a^2 - 1) + \frac{\rho^2}{\sqrt{m}}(c_s^2 - 1) \quad (2)$$

where m is the number of servers in that node.

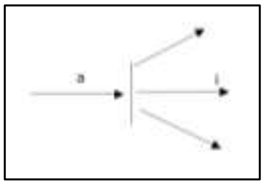

Figure 4: Split network operation

Figure 4 shows a split network operation. The regarding formulations are given by (3) and (41):

$$\lambda_i = p_i \lambda_a \quad (3)$$

$$c_i^2 = p_i c_a^2 + 1 - p_i \quad (4)$$

where p_i is the probability of splitting to the i^{th} route.

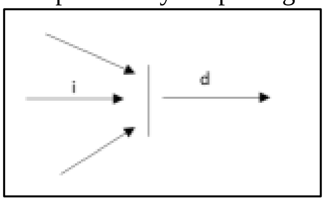

Figure 5: Superposition network operation

Figure 5 shows a superposition network operation. The regarding calculations are given by (5)-(6):

$$\lambda_d = \sum_i \lambda_i \quad (5)$$

$$c_d^2 = \omega \sum_i \left(\frac{\lambda_i}{\sum_k \lambda_k}\right) c_i^2 + 1 - \omega \quad (6)$$

where ω is calculated by (7)-(8):

$$w = [1 + 2.1(1-\rho)^{1.8} v]^{-1} \quad (7)$$

$$v = \left[\sum_i \left({\lambda_i}/{\sum_k \lambda_k}\right)^2\right]^{-1} \quad (8)$$

Note that Poisson process always has $c^2 = 1$. After the calculations of the two moments of the arrivals into the servers, the performance measures can be calculated via *G/G/m* queuing models [13]. For example, the mean waiting time in a queue can be calculated by the below (9) approximation (Whitt 1983b):

$$E(W) \approx \emptyset(\rho, c_a^2, c_s^2, m)\left(\frac{c_a^2 + c_s^2}{2}\right) EW(M/M/m) \quad (9)$$

where $\emptyset(\rho, c_a^2, c_s^2, m)$ is calculated by (10)-(16):

$$\emptyset(\rho, c_a^2, c_s^2, m) = \begin{cases} \left(\frac{4(c_a^2 - c_s^2)}{4c_a^2 - 3c_s^2}\right)\emptyset_1(m,\rho) + \left(\frac{c_s^2}{4c_a^2 - 3c_s^2}\right)\psi\left(\frac{(c_a^2 + c_s^2)}{2}, m, \rho\right), & c_a^2 \geq c_s^2 \\ \left(\frac{c_s^2 - c_a^2}{2c_a^2 + 2c_s^2}\right)\emptyset_3(m,\rho) + \left(\frac{c_s^2 + 3c_a^2}{2c_a^2 + 2c_s^2}\right)\psi\left(\frac{(c_a^2 + c_s^2)}{2}, m, \rho\right), & c_a^2 \leq c_s^2 \end{cases} \quad (10)$$

$$\psi\left(\frac{(c_a^2 + c_s^2)}{2}, m, \rho\right) = \begin{cases} 1, & (c_a^2 + c_s^2)/2 \geq 1 \\ \emptyset_4(m,\rho)^{2(1-(c_a^2+c_s^2)/2)} & 0 \leq (c_a^2 + c_s^2)/2 \leq 1 \end{cases} \quad (11)$$

$$\emptyset_1(m,\rho) = 1 + \gamma(m,\rho) \quad (12)$$

$$\gamma(m,\rho) = min\left\{0.24, (1-\rho)(m-1)((4+5m)^{\frac{1}{2}} - 2)/(16m\rho)\right\} \quad (13)$$

$$\emptyset_2(m,\rho) = 1 - 4\,\gamma(m,\rho) \quad (14)$$

$$\emptyset_3(m,\rho) = \emptyset_2(m,\rho) exp(-2(1-\rho)/3\rho) \quad (15)$$

$$\emptyset_4(m,\rho) = min\{1, (\emptyset_1(m,\rho) + \emptyset_3(m,\rho))/2\} \quad (16)$$

where ρ is the utilization of that node and calculated by (17):

$$\rho = \frac{\lambda}{\mu} \quad (17)$$

The number of transactions in a queue can be calculated by (18):

$$(L) = \lambda \cdot E(W) \quad (18)$$

3.2 Design scenarios for validation of the models

Table 1 shows the design scenarios and their resutls based on the analytical models and the simulation. In the experiments, the acceleration/deceleration delays of lifts ($a_L = d_L$) and shuttles ($a_S = d_S$) are considered to be 1m/sec^2 and 1.5 m/sec^2, respectively. The velocity of lifts (V_L) and shuttles (V_S) are considered to be 1.5 m/sec for both.

Table 1. Design scenarios and their results

			Analytical		Simulation		Error (%100)	
T	$\lambda_S = \lambda_R$ (hour)	ρ_L	$E(W_L)$ (sec.)	$E(L_L)$	$E(W_L)$ (sec.)	$E(L_L)$	$E(W_L)$	$E(L_L)$
20	565	0.95	37.38	11.73	36.56	11.51	2.24	1.91
20	535	0.90	18.57	5.52	16.72	4.98	11.06	10.8
20	505	0.85	12.38	3.47	10.40	2.92	19.04	18.8
19	585	0.95	36.00	11.70	35.09	11.43	2.59	2.36
19	555	0.90	18.23	5.62	16.44	5.08	10.89	10.6
19	525	0.85	12.22	3.57	10.38	3.03	17.73	17.8
18	605	0.95	32.96	11.08	31.66	10.66	4.11	3.94
18	575	0.90	17.42	5.57	15.78	5.05	10.39	10.3
18	545	0.85	11.86	3.59	10.09	3.06	17.54	17.3
17	630	0.95	33.14	11.60	31.64	11.09	4.74	4.60
17	600	0.90	17.50	5.83	15.85	5.29	10.41	10.2
17	565	0.85	11.30	3.55	9.60	3.02	17.71	17.6
16	660	0.95	31.22	11.36	34.64	12.71	9.87	10.6
16	625	0.90	16.96	5.89	15.29	5.32	10.92	10.7
16	585	0.85	11.08	3.63	8.87	2.89	24.92	25.6
15	685	0.95	31.46	11.97	29.92	11.40	5.15	5.00
15	650	0.90	15.85	5.72	14.21	5.14	11.54	11.3
15	615	0.85	10.61	3.63	8.95	3.06	18.55	18.6

In Table 1, $E(W_L)$ and $E(L_L)$ show the expected waiting time of a transaction in a lift queue and the expected number of transactions waiting in a lift queue, respectively. Since, the lifts are bottleneck and the shuttles' utilizations are fairly low in the system, we provide the results solely for lifts. ρ_L denotes the mean utilization of a lift. Since the ρ_L value is exactly same with the simulation result in each scenario, we do not provide the simulation and analytical results separately for this performance measure. The experiments are conducted for different arrival rates of transactions such that we obtain high utilization values for lifts (i.e., 0.95% - 0.85%). Namely, we change the arrival rates to obtain the ρ_L values of 0.95%, 0.90% and 0.85%. The analytical results deviate from the simulation results higher in the lower ρ_L values than the higher ρ_L values. As a note, the deviations are calculated by Absolute Percentage Error (APE) by (19):

$$\frac{\left|E(W)_{Analytical} - E(W)_{\text{Simulation}}\right|}{E(W)_{\text{Simlation}}} \times 100 \quad (19)$$

In the last two columns it can be seen that the errors are typically less than 25.6%. In the higher ρ_L values these errors are reasonably small, i.e. less than 10%. Hence, we could utilize the developed analytical model for performance calculations of the system.

4. CONCLUSION

The aim of this study is to present an open queuing network (OQN) model that can estimate the critical queuing performance measures of a pre-defined SBS/RS design. By the provided model, one would be able to evaluate an SBS/RS' design promptly, by changing those design criteria. The analytical model's results are validated by the simulation results which show reasonably small APE values. As a future study, this work can be extended by considering dual command (DC) scheduling rule in the system.

ACKNOWLEDGMENT

This work was supported by The Scientific and Technological Research Council of Turkey and Slovenian Research Agency: ARRS [grant number: 214M613].

REFERENCES

[1] Borovinšek, M., Ekren, B.Y., Burinskiene, A., Lerher, T. 2016. Multi-objective model for optimization of shuttle-based storage and retrieval systems, Transport, vol. 32 (2), pp.120-137, 2017.

[2] Carlo H.J., I.F.A.Vis, 2012. Sequencing dynamic storage systems with multiple lifts and shuttles. International Journal of Production Economics, 140 844-853.

[3] Ekren, B.Y. 2017. Graph-based solution for performance evaluation of shuttle-based storage and retrieval system. International Journal of Production Research, in press.

[4] Ekren, B.Y., Akpunar, A., Sari, Z., Lerher, T. 2017. A Tool for Time, Variance and Energy Related Performance Estimations in a Shuttle-Based Storage and Retrieval System, Working paper.

[5] Lerher, T., Ekren, B.Y., Sari, Z., Rosi, B. 2016. Method for Evaluating the Throughput Performance of Shuttle Based Storage and Retrieval Systems, Tehnicki Vjesnik – Technical Gazette, vol. 23 (3), pp.715-723.

[6] Lerher, T.; Ekren, B. Y.; Dukic, G.; Rosi, B. 2015a. “Travel time model for shuttle-based storage and retrieval systems.” International Journal of advanced manufacturing Technology 78(9 – 12): 178 - 190.

[7] Lerher, T.; Ekren, Y. B.; Sari, Z.; Rosi, B. 2015b. “Simulation analysis of shuttle based storage and retrieval systems.” International Journal of Simulation Modelling 14(1): 178-190.

[8] Lerher, Tone. 2013. Modern automation in warehousing by using the shuttle based technology. V: ARENT, Doug (editor), FREEBUSH, Monica (editor). Automation Systems of the 21st Century: New Technologies, Applications and Impacts on the Environment & Industrial Processes. New York: Nova Publishers, cop. 51-86.

[9] Marchet, G., Melacini, M., Perotti, S., Tappia, E. 2012. Analytical model to estimate performances of autonomous vehicle storage and retrieval systems for product totes. International Journal of Production Research 51(14) 4365–4387.

[10] Ninga, Z., Lei L., Saipeng Z., Lodewijks G. 2016. An efficient simulation model for rack design in multi-elevator shuttle-based storage and retrieval system. Simulation Modelling Practice and Theory 67, 100–116.

[11] Tappia, E., Roy, D., Koster, R.D., Melacini, M., 2016. Modeling, Analysis, and Design Insights for Shuttle-Based Compact Storage Systems. Transportation Science, in press.

[12] Whitt, W. 1983a. Performance of the Queueing Network Analyzer. Bell System Technical Journal 62(9) 2779-2815.

[13] Whitt, W. 1983b. Approximations for the G/G/m Queue. Production and Operations Management 2 (2) 114-161.

[14] Zou, B., X. Xu, Y.Y. Gong and R. DeKoster. 2016. Modeling parallel movement of lifts and vehicles in tier-captive vehicle-based warehousing systems. European Journal of Operational Research 254 (1) 51–67.

